similar effect as a related change in the pressure gradient parameter β which, although affecting all the profiles, does not affect the relations between profile quantities.

Using the normalized form of the profile quantities, the solutions to the similarity equations at any particular value of S_w and ω therefore provide a set of universal relations that are valid for all S_w and ω. The profile functions applicable for three-dimensional flow may thus be obtained from the Cohen-Reshotko[5] similarity solution for two-dimensional flow (i.e., where $\omega = 0$). This is of considerable gain, as the solutions are readily available and the onerous task of recalculating the profile functions at each value of S_w and ω is avoided.

In order to make the method amenable to numerical techniques, the profile functions thus obtained are finally expressed in the form of polynomial functions of the parameters H and b, using a least squares curve fit.[2,3]

The final form of the governing differential equations becomes

$$\bar{F}\frac{d\ln\delta_i^*}{dx}+\frac{\partial\bar{F}}{\partial H}\frac{dH}{dx}+\frac{\partial\bar{F}}{\partial b}\frac{db}{dx}+\bar{J}\frac{d\ln M_e}{dx}=\frac{B(1+m_e)}{m_e(1+m_\infty)}\frac{\tan(\theta_e-\alpha_w)}{\delta_i^*} \quad (12)$$

$$H\frac{d\ln\delta_i^*}{dx}+\frac{dH}{dx}+(2H+1+S_w\bar{E})\frac{d\ln M_e}{dx}=BC\frac{M_\infty}{M_e}\frac{P}{\delta_i^*Re_{\delta_i^*}} \quad (13)$$

$$J\frac{d\ln\delta_i^*}{dx}+\frac{dJ}{dH}\frac{dH}{dx}+(3J+2S_w\bar{T}^*)\frac{d\ln M_e}{dx}=BC\frac{M_\infty}{M_e}\frac{R}{\delta_i^*Re_{\delta_i^*}} \quad (14)$$

$$T^*\frac{d\ln\delta_i^*}{dx}+\frac{\partial T^*}{\partial H}\frac{dH}{dx}+\frac{\partial T^*}{\partial b}\frac{db}{dx}+T^*\frac{d\ln M_e}{dx}=BC\frac{M_\infty}{M_e}\frac{Q}{\delta_i^*Re_{\delta_i^*}} \quad (15)$$

These equations are similar in form to the equations for two-dimensional flow[3] and the same solution procedure may thus be employed. The model for the outer inviscid flow, which specifies the relationship between the streamline inclination at the edge of the boundary layer θ_e, local static pressure p_e and Mach number M_e can be provided by any of the methods common to inviscid flow theory.

III. Axisymmetric Flow with Spin

The results of the preceding section can be readily generalized to include axisymmetric flow. It is assumed that $\delta \ll r_w$ and the following quantities are redefined

$$g = w/w_w = w/(\Omega r_w), \qquad \omega = w_w{}^2/2h_{0_e}$$

where r_w is the local radius of the body and Ω is the angular velocity of the body.

For k_1 constant, i.e.,

$$\frac{1}{S_w}\frac{dS_w}{dx}=\frac{1}{\omega}\frac{d\omega}{dx}=\frac{2}{r_w}\frac{dr_w}{dx}$$

then

$$g = s \quad (16)$$

and the governing equations reduce to those for three-dimensional plane flow [Eqs. (12–15)] with additional right-hand side terms, respectively,

$$-\left[\frac{2S_w(1+m_e)}{m_e}\bar{E}-\frac{Z}{m_e}\right]\frac{d\ln r_w}{dx} \quad (12')$$

$$-\left[H+\frac{S_w(1+m_e)}{m_e}k_1G\right]\frac{d\ln r_w}{dx} \quad (13')$$

$$-\left[J+\frac{2S_w(1+m_e)}{m_e}k_1W^*\right]\frac{d\ln r_w}{dx} \quad (14')$$

$$-3T^*\frac{d\ln r_w}{dx} \quad (15')$$

The profile functions are unchanged as they are obtained from the similarity solutions with $dr_w/dx = 0$.

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Linear Spatial Stability of the Plane Poiseuille Flow

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THE linear stability of the plane Poiseuille flow has received much attention in both theory and experiment. Many of the investigators[1,2] worked out analytical solutions by assuming that the periodic disturbances were subject to amplification in time. This, however, is not a suitable model for the disturbances investigated experimentally, which are quasi-steady and vary in amplitude with distance downstream. A better model is obtained by considering a disturbance travelling in the direction of flow and having a nondimensional stream function of the form

$$\psi = \phi(y)\,e^{i(\alpha x-\beta t)}+\tilde{\phi}(y)\,e^{-i(\tilde{\alpha}x-\beta t)} \quad (1)$$

where the frequency β is real and the symbol ˜ denotes a complex conjugate. The amplitude of the perturbation $\phi(y)$ is assumed to be small. The substitution of ψ in the nondimensional linearized vorticity equation for the perturbation leads to the Orr-Sommerfeld equation

$$(\bar{u}-\beta/\alpha)(\phi''-\alpha^2\phi)-\bar{u}''\phi+(i/\alpha R)(\phi''''-2\alpha^2\phi+\alpha^4\phi)=0 \quad (2)$$

where an accent (′) indicates differentiation with respect to y, R is the Reynolds number, and $\bar{u} = 1-y^2$ is the nondimensional mean flow velocity. This equation, together with its boundary conditions

$$\phi(y)=0, \quad \phi'(y)=0 \quad \text{at} \quad y=\pm 1 \quad (3)$$

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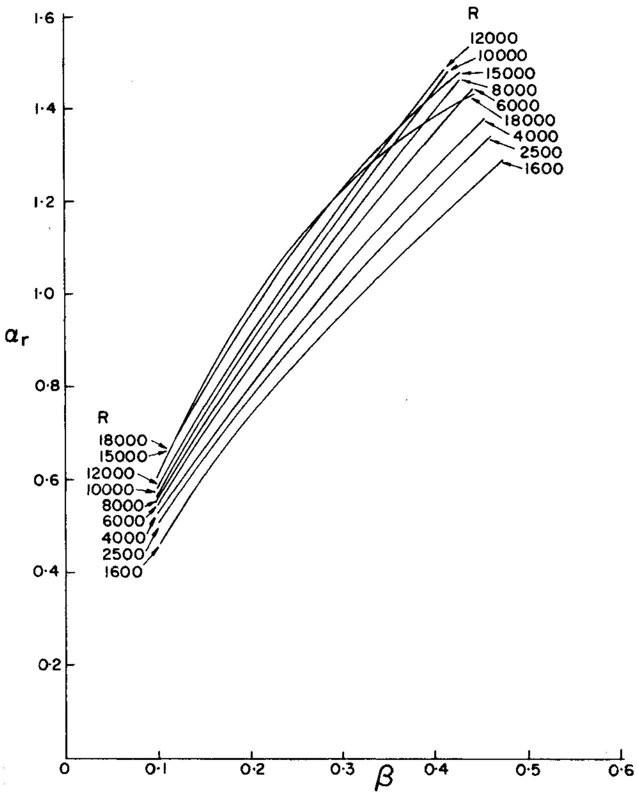

Fig. 1 Variation of α_r with β at constant R.

constitutes a nonlinear eigenvalue problem to determine $\alpha = \alpha_r + i\alpha_i$ as a function of β and R. The given flow will be unstable to disturbances of the form (1) if, for some values of β and R, an eigenvalue can be found with α_i negative. If α_i is put equal to zero in Eq. (2), then it is obvious that the required neutral solutions of Eq. (2) can be found immediately from the neutral ($c_i = 0$) solutions of the temporal case. By imposing the normalizing condition, $\psi(0) = 1$, the neutral solutions of Eq. (2) will be identical with the neutral solutions

Table 1 Parameters on the neutral curve

Reynolds number, R	α_r	β	$d\alpha_{r/d\beta}$	$d\alpha_{i/d\beta}$
18000	1.061	0.2312	3.180	0.9171
15000	1.069	0.2431	3.081	0.7570
12000	1.084	0.2576	2.979	0.6216
10000	1.099	0.2735	2.860	0.5510
8000	1.089	0.2803	2.750	0.3784
6000	1.057	0.2820	2.621	0.1176
5780	1.023	0.2709	2.602	0.0121
5780	1.011	0.2670	2.599	−0.0178
6000	0.997	0.2479	2.660	−0.1530
8000	0.849	0.1960	2.863	−0.3233
10000	0.792	0.1689	3.042	−0.4780
12000	0.758	0.1537	3.184	−0.5468
15000	0.712	0.1341	3.374	−0.6702
18000	0.688	0.1221	3.533	−0.7383

of the time-amplified case[3] (α_r will, of course, be positive for these disturbances).

The objective of this Note is to provide some numerical results for the linear spatial stability of the plane Poiseuille flow. As stated earlier, the mathematical system consisting of Eqs. (2) and (3) is a nonlinear eigenvalue problem which gives the relationship among α, β, and R. The solution of this eigenvalue problem is obtained by first applying the finite-difference techniques to reduce the differential equations to a system of $(N+1)$ homogeneous algebraic equations, where N is the number of equal intervals into which the range $0 \leqq y \leqq 1$ is divided. In order to reduce the truncation error without increasing the order of the difference equation, the Noumerov[4] transformation is used on the derivatives. Then the eigenvalue α is determined by solving the characteristic determinant $F(\alpha, \beta, R) = 0$. To do this, an initial estimate for α is selected; next the value of the determinant is computed by triangularizing the matrix with a Gauss elimination scheme. An iterative procedure, e.g., Muller,[5] is then employed to determine the value of α which produces a sufficiently small value of the determinant, i.e., an eigenvalue.

The primary interest of the present study is to investigate the effect of Reynolds number on the spatial stability of the plane Poiseuille flow. This is best achieved by obtaining the eigenvalues, α_r and α_i, at different frequencies β for Reynolds numbers

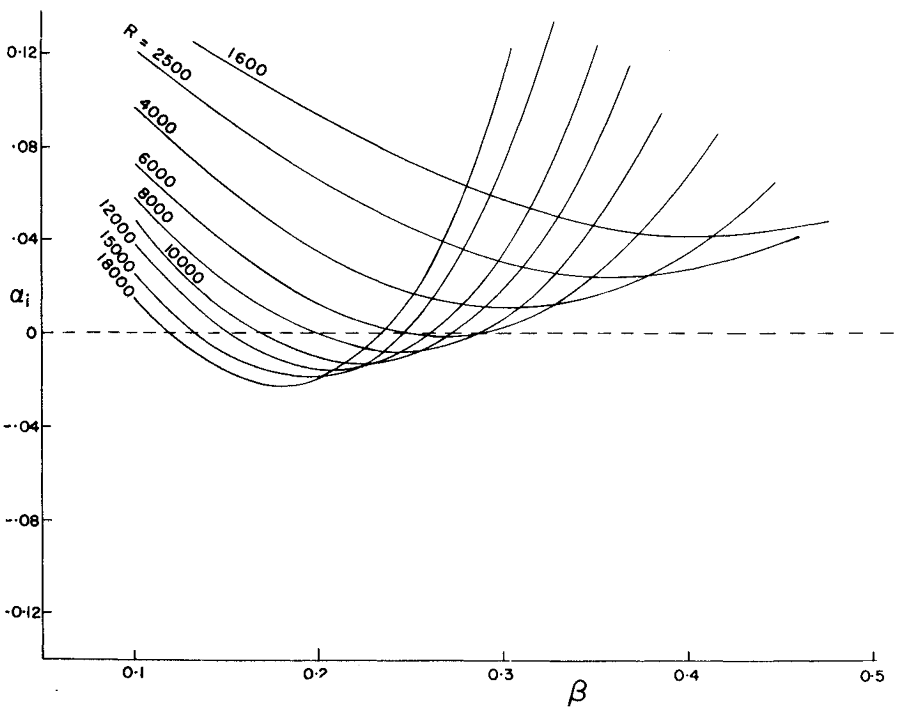

Fig. 2 Variation of α_i with β at constant R.

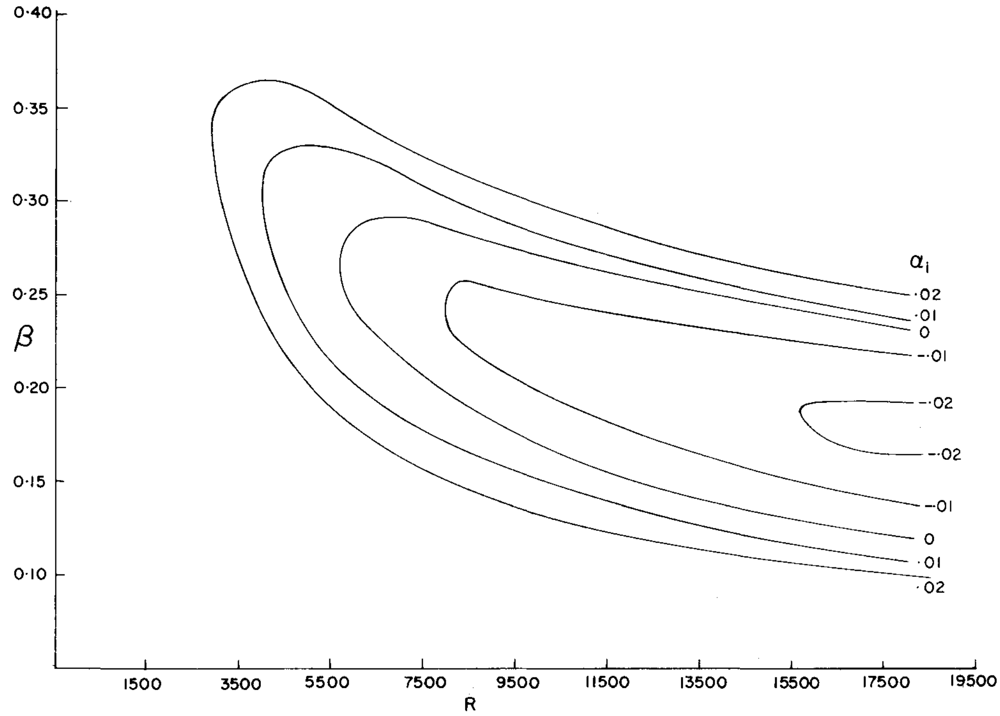

Fig. 3 Variation of β with R at constant α_i.

ranging from 1600 to 18,000. These results are plotted in Figs. 1 and 2 showing α_r and α_i vs the real frequency β. At each value of R the range of values of β was chosen to include the whole region of amplification and also sections of the damping region near the neutral curve. In Fig. 1, the graphs of α_r vs β are monotonically increasing and the slope of these lines, representing the reciprocal of the group velocity,[6] becomes slightly steeper as the Reynolds number increases. However, it is observed that at Reynolds number above 12,000, the slope $d\alpha_r/\partial\beta$, decreases with increasing β at values of β which correspond to points in the damping region beyond those of amplification. For amplification, α_i must be negative and it is evident from Fig. 2 that, according to the present calculations, the critical Reynolds number is slightly less than 6000.

The curves of $\alpha_i = \text{const}$ are shown in Fig. 3. The important parameters of the neutral stability curve are given in Table 1. The critical Reynolds number, according to present calculations, is 5778 and the corresponding value of α_r being 1.0219. The numerical calculations of Thomas[3] give a neutral stability curve which is almost identical with the present results. Thomas[3] found the critical Reynolds number to be 5780 with the corresponding α to be 1.026. Orszag's[7] results of the critical Reynolds number and wave number α were 5772 and 1.0206, respectively. His numerical solution was based on expressing the differential equations in terms of Chebyshev polynomials. The frequency β corresponding to the critical Reynolds number is found to be 0.269.

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Experimental Study of Separation from the Base of a Cone at Supersonic Speeds

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Introduction

FIGURE 1 qualitatively illustrates the nature of the base separation as observed in recent BRL Wind Tunnel tests on the base of a 10° half-angle sting supported cone. At the model corner, the external flow and the cone boundary layer expand through a strong expansion fan. The developing shear layer reattaches on the model sting support where part of the shear layer is recirculated back along the sting. This return flow separates from the sting close to the model and reattaches on the model base. Near the conical trailing edge, the recirculating flow separates again and merges with the shear layer downstream from the model. Apparently a small fraction of the cone surface boundary layer turns the corner and separates some distance from the corner along with the main recirculating flow.

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